

On the topology of untrapped surfaces

István RÁCZ *

RMKI, H-1121 Budapest, Konkoly Thege Miklós út 29-33, Hungary

June 29, 2010

Abstract

Recently a simple proof of the generalizations of Hawking’s black hole topology theorem and its application to topological black holes for higher dimensional ($n \geq 4$) spacetimes was given [14]. By applying the associated new line of argument it is proven here that strictly stable untrapped surfaces possess exactly the same topological properties as strictly stable *marginally outer trapped surfaces* (MOTSs) are known to. In addition, a quasi-local notion of outwards and inwards pointing spacelike directions—applicable to untrapped and marginally trapped surfaces—is also introduced.

PACS number: 04.70.Bw, 04.20.-q

1 Introduction

Hawking’s black hole topology theorem [11] plays a key role in 4-dimensional black hole physics from the beginning of the 70’s. By making use of a variant of Hawking’s argument, almost three decades later, in the late 90’s, Gibbons [9] and Woolgar [15] could also characterize the so-called “topological black hole” spacetimes—to which Hawking’s original argument do not apply—by deriving a genus dependent lower bound for the entropy of these black holes.

Motivated by the considerable increase of interest in higher dimensional black hole configurations of string theory and other generalizations of Einstein theory of gravity, during the last couple of years Galloway and his collaborators (see Refs. [3, 5, 6, 7]) provided important generalizations of Hawking’s [11] black hole topology theorem, and also that of Gibbons’ [9] and Woolgar’s [15] results to higher ($n \geq 4$) dimensional Einstein’s theory of gravity where considerations were restricted exclusively to marginal surfaces, more precisely, to MOTSs.

In our previous paper [14] a simple self-contained proof of these recent generalizations has been derived and it was also proved that these results are valid in a more generic context, i.e., for a much higher variety of theories, than anticipated before. Surprisingly, the applied new line of argument has shown itself to be even more effective. More specifically, according to the

*email: iracz@rmki.kfki.hu

main result of the present paper, the assertion of Theorem 4.1 of [14] remains intact whereas the set of strictly stable MOTSs considered therein is subtended by the set of strictly stable untrapped surfaces. In order to make the significance of this result to be more transparent, let us mention that compact orientable MOTSs with no boundary are expected to represent the boundary of black hole regions [1] while untrapped surfaces of the same type are thought to fill up the entire of the exterior regions. Thereby the untrapped surfaces are, in fact, more common than MOTSs. This becomes to be more manifest if one thinks of the fact that they are also present in spacetimes which do not even have a black hole region.

2 Preliminaries

Since our argument does apply to any metric theory of gravity within this paper, likewise in [14], a spacetime is supposed to be represented by a pair (M, g_{ab}) , where M is an n -dimensional ($n \geq 4$), smooth, paracompact, connected, orientable manifold while g_{ab} is a smooth Lorentzian metric of signature $(-, +, \dots, +)$ on M . We assume that (M, g_{ab}) is time orientable and that a time orientation has been fixed. No use of any sort of field equation concerning the spacetime metric or the matter content will be made. Instead, only the following generalized form of *dominant energy condition* will be applied. As in [14], a spacetime (M, g_{ab}) is said to satisfy the *generalized dominant energy condition* if there exists a smooth real function f on M such that for all future directed timelike vector t^a the combination $-[G^a_b t^b + f t^a]$ is a future directed timelike or null vector, where G_{ab} denotes the Einstein tensor $R_{ab} - \frac{1}{2}g_{ab}R$. It is straightforward to justify (see, e.g., [14] for details) that in Einstein's theory of gravity the generalized dominant energy condition holds, with the choice $f = \Lambda$, if and only if the energy-momentum tensor, T_{ab} , satisfies the standard form of the dominant energy condition.

As indicated above, *untrapped* surfaces of $n \geq 4$ dimensional spacetimes will be at the center of our concern in this paper. In providing their definition start with a smooth orientable $(n - 2)$ -dimensional compact manifold \mathcal{S} with no boundary in an n -dimensional spacetime (M, g_{ab}) . Let ℓ^a and n^a be smooth future and past directed null vector fields on \mathcal{S} , respectively, which are scaled such that $n^a \ell_a = 1$, and that are also normal to \mathcal{S} , i.e., $g_{ab} \ell^a X^b|_{\mathcal{S}} = g_{ab} n^a X^b|_{\mathcal{S}} = 0$ for any vector field X^a tangent to \mathcal{S} . Note that these conditions ensure that neither ℓ^a nor n^a vanishes on \mathcal{S} . Consider then the null hypersurfaces \mathcal{L} and \mathcal{N} generated by geodesics starting on \mathcal{S} with tangent ℓ^a and n^a . By choosing synchronized affine parameterizations to these null geodesic curves the vector fields ℓ^a and n^a extend to \mathcal{L} and \mathcal{N} , respectively. These hypersurfaces are smooth in a neighborhood of \mathcal{S} , and also they are smoothly foliated by the level surfaces—which are $(n - 2)$ -dimensional compact manifolds homologous to \mathcal{S} —of the applied synchronized affine parametrizations. Denote by ϵ_q the volume element associated with the metric, q_{ab} , induced on these $(n - 2)$ -dimensional surfaces. Then the null expansions $\theta^{(\ell)}$ and $\theta^{(n)}$ with respect to ℓ^a and n^a are defined as

$$\mathcal{L}_\ell \epsilon_q = \theta^{(\ell)} \epsilon_q \quad \text{and} \quad \mathcal{L}_n \epsilon_q = \theta^{(n)} \epsilon_q, \quad (2.1)$$

where \mathcal{L}_ℓ and \mathcal{L}_n denote the Lie derivatives with respect to the null vector fields ℓ^a and n^a ,

respectively.

According to the original definition of Penrose [13] a 2-dimensional surface \mathcal{S} , in a 4-dimensional spacetime, is considered to be future or past trapped if both of the future or past directed null geodesic congruences orthogonal to \mathcal{S} are converging at \mathcal{S} , respectively. Thereby, an $(n-2)$ -dimensional surface \mathcal{S} is called to be *trapped* if the null expansions $\theta^{(\ell)}$ and $\theta^{(n)}$ are such that one of them is non-negative while the other is non-positive throughout \mathcal{S} . To separate the limiting case it is assumed that neither of these expansions vanishes identically on \mathcal{S} . Accordingly, an $(n-2)$ -dimensional surface \mathcal{S} is called to be *untrapped* if the null expansions $\theta^{(\ell)}$ and $\theta^{(n)}$ are both non-negative or non-positive throughout \mathcal{S} and neither of them vanishes identically there. In the limiting case, i.e., whenever either of the expansions gets to be identically zero \mathcal{S} is called to be *marginal*.

It is worth keeping in mind that there is a much higher variety of $(n-2)$ -dimensional surfaces than the ones covered by the above three categories. As an immediate example one may think of a surface yielded by a generic deformation of a marginal surface \mathcal{S} , one of the null expansions of which, say $\theta^{(\ell)}$, is identically zero on \mathcal{S} originally, but which may be altered such that $\theta^{(\ell)}$ changes sign after the deformation is performed.

What distinguishes marginal and untrapped surfaces is that a meaningful quasi-local concept of outwards and inwards directions can be associated with these surfaces which—as it immediately follows from the details of the construction below—cannot be done in case of generic $(n-2)$ -dimensional surfaces. In introducing these notions let us consider a marginal or an untrapped surface \mathcal{S} and assume that $\theta^{(\ell)} \geq 0$ and $\theta^{(n)} \geq 0$. [Note that these inequalities may be assumed to be satisfied, without loss of generality, since if they did not hold we could apply the transformation $\ell^a \rightarrow \ell'^a = -\ell^a$ and $n^a \rightarrow n'^a = -n^a$ to ensure them.] Consider now a vector field Z^a that is given by a linear combination of the form $Z^a = A\ell^a + Bn^a$, where the coefficients, A and B , are smooth functions on \mathcal{S} . It is straightforward to see that for any particular choice of A and B the vector field Z^a is smooth and spacelike everywhere on \mathcal{S} whenever A and B are both positive or negative throughout \mathcal{S} .

In order to justify that a meaningful quasi-local concept of outwards and inwards directions may be adequately associated with these type of nowhere vanishing spacelike vector fields fix the coefficients A and B and consider the spacelike geodesics starting on \mathcal{S} with tangent Z^a . Denote by z and \tilde{Z}^a a synchronized affine parameter and the associated tangent field along these geodesics, respectively. Consider then the one-parameter family of surfaces \mathcal{S}_z yielded by the Lie-transport of \mathcal{S} with respect to \tilde{Z}^a . It is said then that the vector field Z^a , defined on \mathcal{S} , points *outwards* or *inwards* if the total variation $\delta_Z \mathcal{A} = \frac{d\mathcal{A}(\mathcal{S}_z)}{dz}|_{z=0}$ of the “area” $\mathcal{A}(\mathcal{S}) = \int_{\mathcal{S}} \epsilon_q$, in the direction of Z^a , is positive or negative, while $\delta_Z \mathcal{A}(\mathcal{S}') \geq 0$ or $\delta_Z \mathcal{A}(\mathcal{S}') \leq 0$ for any subset $\mathcal{S}' \subset \mathcal{S}$, respectively. To see that the above selected spacelike vector fields do always have a definite orientation recall that for any subset $\mathcal{S}' \subset \mathcal{S}$

$$\delta_Z \mathcal{A}(\mathcal{S}') = \int_{\mathcal{S}'} \mathcal{L}_{\tilde{Z}} \epsilon_q = \int_{\mathcal{S}'} [A\theta^{(\ell)} + B\theta^{(n)}] \epsilon_q \quad (2.2)$$

which has the appropriate sign whenever the coefficients A and B are both positive or negative throughout \mathcal{S} , respectively. If both $\theta^{(\ell)}$ and $\theta^{(n)}$ vanish identically on \mathcal{S} the above

quasi-local concept of outwards and inwards directions become degenerate in accordance with the fact that then \mathcal{S} is a minimal surface.

In deriving our main results, likewise in [14], we shall also apply a stability assumption. In formulating it recall first that the null vector fields ℓ^a and n^a on \mathcal{S} are not uniquely determined by the conditions we have imposed. In fact, together with ℓ^a and n^a the null vector fields

$$\ell'^a = e^{-v} \ell^a \quad \text{and} \quad n'^a = e^v n^a \quad (2.3)$$

are also suitable, where $v : \mathcal{S} \rightarrow \mathbb{R}$ is an arbitrary smooth function on \mathcal{S} . It is worth emphasizing, however, that the signs of $\theta^{(\ell)}$ and $\theta^{(n)}$, and, in turn, the notion of trapped, untrapped and marginal surfaces, along with the above defined quasi-local notion of outwards and inwards directions, are intact under such a positive rescaling.

An untrapped surface \mathcal{S} is called to be *strictly stable* if there exists a boost transformation of the form (2.3) such that $(\mathcal{L}_{n'} \theta^{(\ell')} + \theta^{(\ell')} \theta^{(n')})|_{\mathcal{S}} \geq 0$ for the vector fields ℓ'^a and n'^a , and such that the inequality is strict somewhere on \mathcal{S} . In the discussion part we shall return to the interpretation of this—apparently technical but in its fundamental nature purely geometrical—requirement. It is worth mentioning though that, e.g., all the metric spheres of the Minkowski spacetime or that of a Schwarzschild spacetime with radius $r > 2M$ are strictly stable untrapped surfaces according to the above definition.

As it was emphasized earlier our aim in this paper is to provide a topological characterization of strictly stable untrapped surfaces. It is well-known that in this type of characterization of surfaces of dimension $s = n - 2 \geq 3$, in an n -dimensional spacetime, the sign of the Yamabe invariant plays a distinguished role. For instance, whenever it is possible to demonstrate that the Yamabe invariant of \mathcal{S} is positive, as it follows from the remarkable results of Gromov and Lawson [10], \mathcal{S} cannot carry a metric of non-positive sectional curvature which, in turn, raises restrictions on the topology of \mathcal{S} .

In recalling the notion of the Yamabe invariant consider first the conformal class $[q]$ of Riemannian metrics on \mathcal{S} determined by q_{ab} , i.e., $[q]$ consist of those Riemannian metrics \hat{q}_{ab} on \mathcal{S} which can be given as positive function times q_{ab} . Then, the conformal invariant Yamabe constant $Y(\mathcal{S}, [q])$, associated with the conformal class $[q]$, is given as

$$Y(\mathcal{S}, [q]) = \inf_{\hat{q} \in [q]} \frac{\int_{\mathcal{S}} R_{\hat{q}} \epsilon_{\hat{q}}}{\left(\int_{\mathcal{S}} \epsilon_{\hat{q}} \right)^{\frac{s-2}{s}}}, \quad (2.4)$$

where $R_{\hat{q}}$ denotes the scalar curvature associated with the Riemannian metric \hat{q}_{ab} . The Yamabe invariant $\mathcal{Y}(\mathcal{S})$ is defined then as the supremum of the Yamabe constants associated with \mathcal{S} , i.e., $\mathcal{Y}(\mathcal{S}) = \sup_{[q]} Y(\mathcal{S}, [q])$.

It is worth recalling that according to important results of Aubin and Schoen the Yamabe invariant $\mathcal{Y}(\mathcal{S})$ is known to be bounded from above by the Yamabe constant of a sphere of dimension $s = n - 2 \geq 3$ with its standard metric. It also immediately follows from (2.4) that in case of a 2-surface the Yamabe constant reduces to 4π times the Euler characteristic $\chi_{\mathcal{S}}$ of \mathcal{S} , i.e., $Y(\mathcal{S}, [q]) = 4\pi\chi_{\mathcal{S}}$, for any conformal class $[q]$ of Riemannian metrics on \mathcal{S} .

Thereby, in virtue of its definition, the Yamabe invariant itself is also equal to 4π times the Euler characteristic of \mathcal{S} .

3 The main result

Now, by making use of the above recalled notions, our main result is formulated as.

Theorem 3.1 *Let (M, g_{ab}) be a spacetime of dimension $n \geq 4$ in a metric theory of gravity. Assume that the generalized dominant energy condition, with smooth real function $f : M \rightarrow \mathbb{R}$, holds and that \mathcal{S} is a strictly stable untrapped surface in (M, g_{ab}) .*

- (1) *If $f \geq 0$ on \mathcal{S} then \mathcal{S} is of positive Yamabe type, i.e., $\mathcal{Y}(\mathcal{S}) > 0$.*
- (2) *If $\mathcal{Y}(\mathcal{S}) < 0$ and $f_{\min}^{\mathcal{S}} < 0$, where $f_{\min}^{\mathcal{S}}$ denotes the minimal value of f on \mathcal{S} , then*

$$\mathcal{A}(\mathcal{S}) \geq \left(\frac{|\mathcal{Y}(\mathcal{S})|}{2|f_{\min}^{\mathcal{S}}|} \right)^{\frac{n}{2}}. \quad (3.1)$$

Proof: In justifying the above assertions the following generalization of the argument of [14] will be applied. In order to recall some of the basics of the associated simple geometric setup start with the smooth null hypersurface \mathcal{N} spanned by the $(n-2)$ -parameter congruence of null geodesics starting at \mathcal{S} with tangent n^a . Denote by u the affine parameter along these geodesics that is synchronized such that $u = 0$ on \mathcal{S} and by n^a the tangent field $(\partial/\partial u)^a$ on \mathcal{N} . The $u = \text{const}$ cross-sections, \mathcal{S}_u , provides then a smooth foliation of \mathcal{N} . Denote by ℓ^a the unique future directed null vector field on \mathcal{N} defined by requiring that $g_{ab}n^a\ell^b = 1$, and that ℓ^a is orthogonal to each \mathcal{S}_u . Choose r to be the affine parameter of the null geodesics determined by ℓ^a which are synchronized such that $r = 0$ on \mathcal{N} .

Since ℓ^a is, by construction, smooth on \mathcal{N} the null geodesics starting with tangent ℓ^a on \mathcal{N} do not meet within a sufficiently small open “elementary spacetime neighborhood” \mathcal{O} of \mathcal{S} . The function u is extended then from \mathcal{N} onto \mathcal{O} by requiring its value to be constant along the geodesics with tangent ℓ^a . Then the vector fields n^a and ℓ^a , defined so far only on \mathcal{N} , do also extend onto \mathcal{O} such that the relations $n^a = (\partial/\partial u)^a$ and $\ell^a = (\partial/\partial r)^a$ hold there which immediately guarantee that n^a and ℓ^a commute on \mathcal{O} . The elementary spacetime neighborhood \mathcal{O} is smoothly foliated then by the 2-parameter family of $(n-2)$ -dimensional $u = \text{const}$, $r = \text{const}$ level surfaces $\mathcal{S}_{u,r}$, furthermore, the spacetime metric in \mathcal{O} takes the form

$$g_{ab} = 2 \left(\nabla_{(a} r - r \alpha \nabla_{(a} u - r \beta_{(a} \right) \nabla_{b)} u + \gamma_{ab}, \quad (3.2)$$

where α , β_a and γ_{ab} are smooth fields on \mathcal{O} such that β_a and γ_{ab} are orthogonal to n^a and ℓ^a [12].

Then, by making use of this simple geometrical setup—in particular, equations (3.3), (3.5), (3.6) and (3.7) of [14] such that all the terms proportional to $\theta^{(\ell)}$ are retained—the

relation

$$\mathcal{L}_n \theta^{(\ell)}|_{\mathcal{S}} = G_{ab} n^a \ell^b - \alpha \theta^{(\ell)} - \theta^{(\ell)} \theta^{(n)} + \frac{1}{2} \left[R_q + D^a \beta_a - \frac{1}{2} \beta^a \beta_a \right] \quad (3.3)$$

can be deduced.

In proceeding note first that, according to the following lemma, the second term on the right hand side of the previous equation drops out.

Lemma 3.1 *The metric function α vanishes on \mathcal{N} .*

Proof: In virtue of (3.2) we have that $n^a n_a = -2r\alpha$, which, in particular, implies that the relation

$$\ell^e \nabla_e (n^a n_a) = \partial_r (-2r\alpha) = -2\alpha \quad (3.4)$$

is satisfied on \mathcal{N} , i.e., whenever $r = 0$. The assertion of our lemma follows then from the fact that

$$\ell^e \nabla_e (n^a n_a) = 2n_a \ell^e \nabla_e n^a = 2n_a n^e \nabla_e \ell^a = -2\ell_a n^e \nabla_e n^a = 0, \quad (3.5)$$

holds on \mathcal{N} , where $[n, \ell]^a = 0$ and $n_a \ell^a = 1$, along with the fact that u was chosen to be an affine parameter along the generators of \mathcal{N} , have been applied. \square

In addition, since $-n^a$ and ℓ^a are both future directed null vector fields on \mathcal{S} , and also the generalized dominant energy condition holds the inequality $G_{ab} n^a \ell^b + f \leq 0$ is satisfied on \mathcal{S} . Finally, recall that \mathcal{S} was assumed to be a strictly stable untrapped surface which ensures that the null normals n^a and ℓ^a may be assumed, without loss of generality, to be such that $(\mathcal{L}_n \theta^{(\ell)} + \theta^{(\ell)} \theta^{(n)})|_{\mathcal{S}} \geq 0$, and also that $\mathcal{L}_n \theta^{(\ell)} + \theta^{(\ell)} \theta^{(n)} > 0$ somewhere on \mathcal{S} .

Consequently, whenever \mathcal{S} is a strictly stable untrapped surface and the generalized dominant energy condition is also satisfied then, in virtue of (3.3), the inequality

$$R_q + D^a \beta_a - \frac{1}{2} \beta^a \beta_a \geq 2f \quad (3.6)$$

holds, so that it is strict somewhere on \mathcal{S} . Since (3.6) possesses exactly the same form as (3.12) in [14] from this point the assertions of Theorem 3.1 may be justified simply by repeating the corresponding part of the argument of Section 3 of [14]. \square

4 Discussion

Let us return now to the interpretation of the stability condition we have applied. To this end note first that the second variation $\delta_n \delta_\ell \mathcal{A} = \frac{\partial^2 \mathcal{A}(\mathcal{S}_{u,r})}{\partial u \partial r}|_{u=0, r=0}$ of the area in the principal null directions ℓ^a and n^a reads as

$$\delta_n \delta_\ell \mathcal{A} = \int_{\mathcal{S}} \mathcal{L}_n \mathcal{L}_\ell \epsilon_q = \int_{\mathcal{S}} \left[\mathcal{L}_n \theta^{(\ell)} + \theta^{(\ell)} \theta^{(n)} \right] \epsilon_q. \quad (4.1)$$

Note that $\delta_n \delta_\ell \mathcal{A} = \delta_{-n} \delta_{-\ell} \mathcal{A}$ and that $\delta_n \delta_\ell \mathcal{A} = \delta_\ell \delta_n \mathcal{A}$, where the first equality is trivial while the second one follows from fact that ℓ^a and n^a commute in \mathcal{O} . According to the above observations our strict stability condition is equivalent to the existence of principal null vector fields ℓ^a and n^a on \mathcal{S} such that the second variation $\delta_n \delta_\ell \mathcal{A}$ is positive while $\delta_n \delta_\ell \mathcal{A}(\mathcal{S}') \geq 0$ for any portion $\mathcal{S}' \subset \mathcal{S}$. Note that the strict stability condition used in [1, 2, 3, 5, 6, 7, 14], only in the context of MOTSs, can also be seen to be equivalent to these requirements.

To justify our last claim and also to provide another characterization of the applied strict stability condition recall that under the rescaling (2.3) of the vector fields ℓ^a and n^a on \mathcal{S} the metric function α and the form field β_a transform as $\alpha \rightarrow \alpha' = \alpha e^v$ and $\beta_a \rightarrow \beta'_a = \beta_a + D_a v$. Introducing then the notation $\psi = e^{-2v}$ and $s_a = \frac{1}{2}\beta_a$, it can be verified (see also [14] for more details) that (3.3) takes the form

$$([\mathcal{L}_{n'} \theta^{(\ell')} + \theta^{(\ell')} \theta^{(n')}] \psi)|_{\mathcal{S}} = -D^a D_a \psi + 2s^a D_a \psi + \left[\frac{1}{2} R_q + G_{ab} n^a \ell^b + D^a s_a - s^a s_a \right] \psi, \quad (4.2)$$

where the vanishing of α on \mathcal{S} has been applied. By choosing then “the variation vector field v^a ” to be n^a the right hand side of (4.2) coincides with the action of the “stability operator” L_v , defined by equation (5) of [2], on ψ . Thereby the right hand side of (4.2) determines a linear elliptic operator of the form given by equation (10) of [2], and whence the arguments of Sections 4 and 5 of [2] can also be applied to the present case. Accordingly, \mathcal{S} is a strictly stable untrapped surface iff there exists a function $\psi \geq 0$, $\psi \not\equiv 0$ on \mathcal{S} such that $L_v \psi \geq 0$, $L_v \psi \not\equiv 0$ on \mathcal{S} , or equivalently iff the principal eigenvalue of L_v is positive.

As the domain of outer communication (DOC) of a black hole spacetime is filled up with untrapped surfaces our main result has to have some connection with the topological censorship theorems which provide a topological characterization of DOCs of asymptotically flat or asymptotically locally anti-de Sitter spacetimes (see, e.g., [4, 8]). To indicate the most important differences recall that the topological censorship theorems—that are also known (see, e.g., [8]) to be valid in arbitrary dimension $n \geq 3$ —are inherently global. They assert that the topology of a DOC is *simple* if the topology of the asymptotic region is simple. As opposed to this our result is fully quasi-local. Thereby, it applies to untrapped surfaces regardless whether the underlying spacetime possesses an asymptotic region or not. Nevertheless, it would be useful, not only in exploring some of the deeper connections, to find out whether a foliation of DOCs by strictly stable untrapped surfaces is always possible.

Finally, we would like to emphasize again that in virtue of Theorem 3.1 and Theorem 4.1 of [14] strictly stable MOTSs and untrapped surfaces do possess the same topological properties whenever both type of surfaces exist in a spacetime. In the particular case of a 4-dimensional spacetime in Einstein’s theory of gravity with zero cosmological constant and with matter satisfying the dominant energy condition strictly stable untrapped surfaces must possess the topology of 2-spheres regardless whether the underlying spacetime contains a black hole region or not. Nevertheless, it is also important to keep in mind that since no use of field equations has been made anywhere in our analysis and the spacetime dimension

has also been kept arbitrary the main results of this paper applies to any metric theory of gravity, with dimension $n \geq 4$.

Acknowledgments

This research was supported in part by OTKA grant K67942.

References

- [1] L. Andersson, M. Mars, and W. Simon: *Local existence of dynamical and trapping horizons*, Phys. Rev. Lett. **95**, 111102 (2005)
- [2] L. Andersson, M. Mars, and W. Simon: *Stability of marginally outer trapped surfaces and existence of marginally outer trapped tubes*, arXiv:0704.2889
- [3] M. Cai and G. J. Galloway: *On the topology and area of higher-dimensional black holes*, Class. Quant. Grav. **18**, 2707-2718 (2001)
- [4] J.L. Friedman, K. Schleich and D.M. Witt: *Topological Censorship*, Phys. Rev. Lett. **71**, 1486-1489 (1993); Erratum-ibid. **75** 1872 (1995)
- [5] G.J. Galloway and R. Schoen: *A generalization of Hawking's black hole topology theorem to higher dimensions*, Commun. Math. Phys. **266**, 571-576 (2006)
- [6] G.J. Galloway: *Rigidity of outer horizons and the topology of black holes*, Commun. Anal. Geom. **16**, 217-229 (2008)
- [7] G.J. Galloway and N. O'Murchadha: *Some remarks on the size of bodies and black holes*, Class. Quant. Grav. **25**, 105009 (2008)
- [8] G.J. Galloway, K. Schleich, D.M. Witt and E. Woolgar: *The AdS/CFT correspondence conjecture and topological censorship*, Phys. Lett. B **505**, 255-262 (2001)
- [9] G.W. Gibbons: Class. Quant. Grav. **16**, 1677-1687 (1999) *Some Comments on Gravitational Entropy and the Inverse Mean Curvature Flow*,
- [10] M. Gromov and H.B. Lawson: *Positive Scalar Curvature and the Dirac Operator on Complete Riemannian Manifolds*, Publ. Math. IHES **58**, 83-196 (1983);
- [11] S.W. Hawking: *Black holes in general relativity*, Commun. Math. Phys. **25**, 152-166 (1972)
- [12] S. Hollands, A. Ishibashi and R.M. Wald: *A higher dimensional stationary rotating black hole must be axisymmetric*, Commun. Math. Phys. **271**, 699-722 (2007)

- [13] R. Penrose: *Gravitational collapse and space-time singularities*, Phys. Rev. Lett. **14** 54-59 (1965)
- [14] I. Rácz: *A simple proof of the recent generalizations of Hawking's black hole topology theorem*, Class. Quant. Grav. **25**, 162001 (2008)
- [15] E. Woolgar: *Bounded area theorems for higher-genus black holes*, Class. Quant. Grav. **16**, 3005-3012 (1999)